

**R. H. Enns, B. L. Jones, R. M. Miura, and S. S. Rangnekar (eds.): Nonlinear Phenomena in Physics and Biology. Proceedings of a NATO Advanced Study Institute at the Banff Center, Banff, Alberta, Canada 1980.** New York: Plenum Press 1981, 609 pp., price \$75.00

At first glance present day theoretical chemistry appears as a kingdom of linearity. The impressive success story of numerical quantum chemistry would, in fact, hardly be imaginable without the formal ease of linear variations of linear energy functionals. But does linearity always tell the whole story? Should one ignore that some of the most interesting actual research problems are nonlinear in nature and that the nonlinear models involved often span several disciplines, a simple example being the Volterra-type models in population dynamics which have their analogue in nonlinear optics and plasma physics, in biological competition and selection at the molecular level? The bible, indeed, "does not state that the ultimate fundamental theories in physics should be linear" Fermi is reported to have said. Couldn't this be true on a less fundamental level as well?

Nonlinear theories may be interesting, but nonlinear mathematics is notoriously difficult. The present volume gives an impression how much the state of this art has improved in the last decades. It contains the proceedings of a conference on nonlinear wave phenomena (5 articles), on nonlinear behavior of systems far from equilibrium (5 articles), and on competition and evolution in biophysical systems (2 articles). Though most of the applications are concerned with biological questions and though chemistry does not appear in the title, most of the articles allude to chemical problems and three of the contributions are predominantly dedicated to them: "Bifurcations, fluctuations and dissipative structures" by G. Nicolis, "Chemical oscillations" by L. N. Howard, and "Selection and evolution in molecular systems" by P. Schuster.

The book starts with an introduction to nonlinear waves which after a discourse on the earlier history of the solitary wave concept describes the post-war development beginning with the famous Fermi–Pasta–Ulam study on the dynamics of energy equipartition in systems of masses coupled by weakly nonlinear springs (it was one of the first scientific applications of digital computers). Instead of equipartition Fermi, Pasta and Ulam observed that the energy returned almost entirely to the fundamental mode after appearing at only a few other modes. There was no tendency of thermalization. It was discovered in the sixties that the spring system is equivalent to a discrete approximation of a Korteweg–de Vries equation, originally designed for the description of nonlinear wave propagation in water, which can be transformed into a linear (!) time-dependent Schrödinger equation by a Bäcklund transformation. This opened continents. Today there are numerous applications of solitary waves in particle physics, solid state physics, neurodynamics, and many other fields. A highly speculative hypothesis by Davydov may be of special interest to theoretical chemists. It claims that the energy transfer in alpha-helix-proteins occurs via solitary waves caused by nonlinearities in the potential energy surface of the hydrogen bonds.

Bifurcation theory is a second field of nonlinear mathematics ready for use. After the publication of Sattinger's book on group theoretical methods parts of bifurcation analysis have definitely become routine. Amongst the applications of chemical interest one finds oscillatory reactions, pattern formation by diffusion-coupled reactions and the spontaneous generation of optical activity, to name just a few.

Is linearity itself a unifying concept of theory? I am inclined to say no. The undeniable limitations of linearity, on the other hand, make it necessary to go beyond to the study of nonlinear problems. "It may be that every such problem is individual and requires individual methods. Yet there are definitely some common features and therefore one can learn by comparing different nonlinear problems" Heisenberg remarked in his opening address to the 1968 school on Nonlinear Mathematics and Physics. Theoretical chemistry cannot afford to be afraid of nonlinearity. Some of its major open problems are intimately connected with nonlinearity: the development of reduced descriptions of complex molecules as well as the foundation of an essentially irreversible thermodynamics. The present book contains some excellent survey papers and many valuable references. It may serve as a starting point for a first approach.

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